



Begin by adding in the auxiliary lines FG, FH, FB and FI. Of course, triangle FGH is isosceles with $FG = FH$. Furthermore, since $CB=x$, $FG = FH = \frac{x}{2}$. Also notice that $FB = \frac{x\sqrt{2}}{2}$ (why?).

Consider right triangle FIB. Angle FBI = Angle FBA – Angle DBA = $45^\circ - \arctan\left(\frac{AD}{AB}\right)$ where AB is held constant and $AD = x$. Therefore, Angle FBI = $45^\circ - \arctan\left(\frac{x}{AB}\right)$. By right triangle trigonometry, $FI = FB \cdot \sin(\angle FBI) = \frac{x\sqrt{2}}{2} \cdot \sin\left(45^\circ - \arctan\left(\frac{x}{AB}\right)\right)$.

By the Pythagorean Theorem, $GI = \sqrt{FG^2 - FI^2}$, therefore,

$$GI = \sqrt{\left(\frac{x}{2}\right)^2 - \left(\frac{x\sqrt{2}}{2} \cdot \sin\left(45^\circ - \arctan\left(\frac{x}{AB}\right)\right)\right)^2}$$

and $GH = 2 GI$, so

$$GH = 2 \cdot \sqrt{\left(\frac{x}{2}\right)^2 - \left(\frac{x\sqrt{2}}{2} \cdot \sin\left(45^\circ - \arctan\left(\frac{x}{AB}\right)\right)\right)^2}$$